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Examining children's opportunity to learn mental calculation

Mike Askew
Tamara Bibby
Margaret Brown
Jeremy Hodgen

of King's College, University of London

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Contents

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BEAM Education
Maze Workshops
72a Southgate Road
London N1 3JT

tel: 020 7684 3323
fax: 020 7684 3334
email: info@beam.co.uk

www.beam.co.uk

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Introduction

This project set out to further our understanding, both practical and theoretical, of a number of key issues and questions in the teaching of mental calculation.

Our objectives

1. to review the understandings and interpretations of mental calculation that teachers are developing and that underpin their practices
2. to look at the range of practices in teaching mental calculation that teachers are developing in the light of the National Numeracy Strategy (NNS)
3. to examine the balance teachers attempt to achieve between children recalling number facts and developing strategies for effective mental calculation
4. to see how effective teachers are in going beyond the use of routines and artefacts (for example, number lines) demonstrated in NNS policy documents and training videos to appreciate the principles underpinning such techniques and so bring about shifts in their understanding and practice more generally
5. to define what ‘best practice’ in mental calculation might look like
6. to review insights into pupils’ responses to the range of practices
7. to examine the policy implications from all the above for developing training packages.

With the exception of point 7, all the points in this list of objectives might be considered aspects of ‘opportunity to learn’. ‘Opportunity to learn’ is consistently identified in international studies as a key indicator of pupil attainment. If particular mathematical content is emphasised in curriculum documents and then translated into classroom practice by teachers, pupil attainment in that content area is generally higher.

We can therefore summarise our study as an attempt to explore aspects of ‘opportunity to learn mental calculation’.

We made the concept of ‘opportunity to learn’ operational by examining both quantitative and qualitative aspects of the teaching of mental calculation. In this paper we address two main aspects of ‘opportunity to learn mental calculation’. Firstly, what are the expectations in terms of the amount of teaching time to be given over to mental calculation and what did our quantitative evidence suggest was happening in classrooms? Secondly, what are the curriculum emphases for the teaching of mental calculation and what insights do our qualitative data provide into the ways these are interpreted and implemented?

Project framework

Phase 1

Phase 1 involved the careful selection of LEAs, schools and teachers, followed by planned observations. In selecting schools, we first of all sought advice from LEA inspectors about schools that had been actively involved in local NNS initiatives. Following this, personal contacts and schools' positions within OfSTED inspection cycles helped to identify schools prepared to be involved.

The three LEAs and the six schools[1] (two from each LEA) were chosen to provide some contrast with each other: Northside and Southside LEAs are both inner-city locales, while MarketTown LEA is a small town which provides a commercial hub for local communities; Northside and Southside LEAs score highly on indices of social deprivation, while MarketTown is a relatively affluent LEA, and the two schools selected there have a large proportion of highly educated parents. We decided to concentrate on Years 1, 3, and 5 to avoid the years of compulsory testing.

The teachers we selected had a range of experiences and personal interests – some more interested and involved in mathematics teaching than others. We decided to include NQTs, because these are the group of teachers who only know the National Numeracy Strategy and whose training in college would have been based around Strategy policy imperatives.

Phase 2

Our choice of which three schools and nine teachers to follow up in Phase 2 was based on our observations and discussions in Phase 1. We selected those schools and teachers who had been most interested in the research and whose circumstances (institutional and individual) gave us a range of opportunities.

The final data set complies with our original intentions. While we are also making use of an existing data set (from the Leverhulme Numeracy Research Project[2]) to validate our findings, this paper reports on the data collected within the timescale of this project. The appendices contain a full listing of the teachers, their experience, and the data we collected on each.

Observation

During our research we observed 59 lessons (19 in Key Stage 1 and 40 in Key Stage 2). Each one was followed by a brief discussion with the class teacher, to clarify the aims and perceived outcomes of the lessons. As our purpose was to consider the teaching of mental calculation, we asked to observe lessons where number and calculation was to be the main focus. However, there were some lessons which had other foci: 15 (25%) focused on place value, 3 (5%) focused on shape and space, 3 (5%) focused on data handling and 6 (10%) focused on measures. But even within these lessons with other foci, just over half included work on calculation, most notably in mental and oral starters.

1. The names of all LEAs, schools and teachers have been changed for the purpose of this paper.

2. For more information go to www.kcl.ac.uk/depsta/education/research/leverhul.html

During observations we took fieldnotes and wrote these up immediately afterwards. The focus throughout was on the opportunities to learn and the use children were able to make of these: we observed teaching and the sense the children were making of that teaching. While teachers were talking to the whole class, notes were taken which included timings and copies of anything the teacher wrote up for all the class to consider. While children worked, we moved around the classroom talking to individuals, unless the teacher had asked us to work with a group or individual. During those times when a teacher worked with a small group, we observed that teaching. During both Phases, we kept copies of teachers' planning and any worksheets or textbook pages used. Following lessons (particularly in Phase 2) we took photocopies of a selection of children's work. We used audio tape to record mental and oral starters and plenaries, as there tended to be more teacher talk here and parts were transcribed to fill in detail in the fieldnotes. Interviews and discussions that followed the lessons were also taped and transcribed for later analysis.

Lesson analysis

We coded each lesson description for mathematical content. As the different parts of a lesson (the mental and oral starter, the main activity and the plenary) often had different emphases, we coded each of these separately. Also, some parts of a lesson contained different, but closely linked foci – for example, a main activity might contain work on place value and subtraction. Where this was the case, we gave that part of the lesson more than one subject code. In other words, we tried to capture, in as much detail as possible, all the times when opportunities for dealing with mental calculation arose in a lesson.

The Framework[3] emphasises the use of mental calculation to support the development of written methods, so we have not drawn a clear boundary between these forms of working.

The qualitative analysis of fieldnotes was undertaken iteratively, to provide for consistent and ongoing reflection. We coded interactions looking at who had initiated the interaction, the kind of question (closed, open, directed at an individual, group or whole class), and the kind of response (one word, longer statements, further questions). We considered the ways in which resources were used (the degree of prescription or flexibility allowed) and the areas of mathematics being engaged with. We compared the teachers' intentions (from discussion and interviews) with the lessons themselves and the Strategy guidelines in the Framework.

From these analyses critical incidents emerged; we considered these in more detail and compared them with each other. An example of one such incident is given on page 13.

Key themes emerged: the treatment of particular aspects of mathematics; the use of representations (pictures, models); the use of resources; issues of authority; and the space created (or closed down) within lessons for exploration and discussion. We are continuing to explore these themes and will write about them in future publications.

3. Department for Education and Skills *Framework for teaching mathematics from Reception to Year 6* DfES 1999, referred to in this paper as 'the Framework'.

Main findings

Lesson timings (recommended)

Although the National Curriculum for Mathematics, England's mandatory curriculum document, does not specify any expected times for curriculum coverage, the Framework does give indicative timings, for mathematics lessons, their constituent parts and curriculum coverage.

The Strategy recommends that Key Stage 1 pupils have a three-part daily mathematics lesson of around 45 minutes and Key Stage 2 pupils a lesson lasting 50 to 60 minutes. The Strategy also makes recommendations about the length of time that should be spent on each of the three parts of a lesson:

mental and oral starter	5 – 10 minutes
main activity	30 – 40 minutes
plenary	10 – 15 minutes

Lesson timings (observed)

The lessons we observed were predominantly within the timeframes suggested for the daily mathematics lesson. The shortest lesson that we observed was with a Year 5 class and lasted 25 minutes. The longest lesson observed lasted 90 minutes and was with Year 3. The average lesson length was 57 minutes. The overall length of lessons, twenty (34%) of which were over an hour, tended to be dictated by timetabling considerations (assemblies and play times) rather than policy.

Most of the lessons we observed conformed broadly to the 'three-part' structure recommended by the Strategy, although in six (10%) of the lessons there was no plenary. But within this three-part format, a striking feature of the lessons was the balance of time. Broadly, the mental and oral starters were long whilst the plenaries were short. In total twelve (20%) of the lessons we observed had a mental/oral starter of 20 minutes or more, whilst 27 (45%) of the plenaries were 5 minutes or less. In comparison only two (3%) starters were 5 minutes or shorter and only two plenaries (3%) were longer than 15 minutes. Rather than generally being too long or too short, there was a wide spread of variation from the 30-40 minute recommendation for the main part of the lesson: eleven (19%) of the main activities were shorter than 25 minutes and eight (14%) were longer than 45 minutes.

Balance of addition/subtraction and multiplication/division (recommended)

There is no statutory guidance on the balance of time that should be spent on particular topics within the mathematics curriculum. However, the Strategy was established in the context of a belief that teachers were not allocating enough time to calculation, and it produced medium-term plans for schools to use that were based on this assumption. The schools we visited were all using these medium-term plans to guide their own more detailed planning. Using these same medium-term plans from the Strategy, we calculated roughly what proportion of lessons pupils might be expected to spend on the topics within calculation (see Table 1). Note that the Framework does not include any work on multiplication or division in Year 1, with a compensatory shift in the direction of multiplication and division in later years.

Proportion of time on topics (expected)

	Addition and subtraction	Multiplication and division
Year 1	100%	0
Year 3	64%	36%
Year 5	40%	60%

Table 1

Balance of addition/subtraction and multiplication/division (observed)

Table 2 is based on the information in Table 1; we compared the lesson elements that we observed with the number that we might have expected to see, and from that extrapolated a matching set of percentages. Of course, differences between observed and expected lesson content may be the result of the timings of visits, rather than emphases placed by the schools and teachers on different curriculum elements. We need to treat this comparison with some caution, as we did not theoretically sample the lessons we saw.

Proportion of time on topics (observed)

	Addition and subtraction	Multiplication and division
Year 1	100%	0
Year 3	62%	38%
Year 5	56%	44%

Table 2

Amongst the detail of our observations, we noticed that in Year 5 only one lesson included use of electronic calculators; we had expected eleven lessons to be calculator-based, as 20% of Year 5 lessons on calculation are supposed to make use of calculators.

In summary, our findings indicated that, while the balance of teaching in Years 1 and 3 was broadly in line with Strategy recommendations, there was no real evidence to show a shift in favour of multiplication and division in Year 5 – as one would have expected.

Emphasis on strategic thinking (recommended)

A key aspect of the Numeracy Strategy is the encouragement of strategic thinking; when the Strategy was first introduced, this was flagged up as a major change from previous teaching. All the materials used by Strategy consultants in LEA- and school-based training stressed that the first aim of the Numeracy Strategy was 'an increased emphasis on mental calculation', as did the self-study materials sent into schools.

As part of this increased emphasis, the Strategy stresses the importance of pupils developing a 'repertoire' of mental and written calculation strategies from the earliest years, along with an ability to select between these according to the size of the numbers and the purposes of the calculation.

> Not everyone does a mental calculation like 81 – 26 in the same way (nor is it necessary for them to do so) but some methods are more efficient and reliable than others. By explaining, discussing and comparing different … methods, you can guide pupils towards choosing the methods which are most efficient and which can be applied generally.[4]

An important aspect of strategic thinking is the development of informed decision-making. And throughout the Framework decision-making is included as a skill to be developed

> Pupils should be taught to choose and use appropriate number operations and ways of calculating to solve problems.
>
> As outcomes Year 1 pupils should, for example: … decide whether the calculation can be done mentally or needs the use of apparatus such as counters, cubes … numbered square or number track.[5]

Emphasis on strategic thinking (observed)

Of the 59 lessons we observed, only one Year 1 lesson listed 'choose and use appropriate number operations' as its objective. Interestingly, this is only half of the objective in the Framework; it continues '… and ways of calculating to solve problems'[6]. In the lesson itself, it was clear that there were no decisions to be made, although there were right and wrong answers to be identified on a worksheet that included questions such as 3 □ 2 = 5 and 8 □ 4 = 4.

Our analysis of the Framework suggests that, while decision-making is central to the philosophy of the Numeracy Strategy, it is not at the heart of the objectives – so perhaps this omission in lesson plans is unsurprising.

As the lesson objectives did not indicate strategic work, we had to make coding decisions about which lesson elements were strategic. Looking at the different kinds of activity in each lesson element, we coded the data to consider whether recall, procedural or strategic thinking was being encouraged. (We defined

4. *Framework*, op. cit. p7
5. Ibid. p60
6. Ibid.

procedural lessons as those that lacked a quality of informed decision-making and focused on the use of a specific method without discussing why this method was appropriate.) **This coding required us to look not simply at the ideas being introduced but at how the teacher was introducing those ideas.**

In the majority of instances observed that included use of the empty number line, for example, the teaching failed to raise issues of efficient and different ways of using it. And yet the empty number line is a valuable pedagogic tool for exploring with children counting-on or -back methods for addition and subtraction. One Year 3 teacher insisted that the children make their jumps on the number line be of 1, 5 or 10 (as she felt these were numbers the children could count in easily); she stopped a child using a jump of 4 to reach the next 'ten', which would have resulted in an efficient strategy.

Again, a Year 1 lesson where the teacher was teaching her class to count on by putting the larger number in their heads was coded as procedural. While such a method might be deemed to be strategic, in this lesson there was no discussion about why the larger (rather than the smaller or the first) number should be put in the head. During the observation, asking children in the class why they thought they were doing this provoked answers indicating that the children were only doing this because the teacher had told them to.

On the basis of this coding, we saw no lessons that could be classified as predominantly strategic. However, we coded lesson elements as strategic where at least one incident was judged to have allowed children to question decision-making processes.

An example from a Year 5 plenary involved a teacher responding positively to a child challenging an assertion that 'you can't take a big number from a smaller one' and working through the subtraction with negative numbers in the intermediate steps. (This example is discussed in more detail below.) We also coded as strategic those lessons where multiple methods were being highlighted, even if there were no explicit comparison of those methods (indeed, of the 175 elements coded, only one mental and oral starter contained any discussion of the relative efficiency of different methods).

It is important to note that in none of the lesson elements we observed was there more than one incident of strategic behaviour, so this coding might be considered generous. The largest part of all the lessons we saw was procedural.

Work on developing recall took place mainly in mental and oral starters, with some work in plenaries. Only 5 of the 46 elements coded as recall took place in plenaries; of these, 4 were associated with addition and subtraction.
No work developing recall was seen in the main activities. When work on recall took place in mental and oral starters, it was generally handled in a procedural manner, although occasionally a more strategic line was taken.

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Summary data

Tables 3 and 4 show the different emphases on recall in mental and oral starters.

Tables 5 and 6 indicate the coding of procedural and strategic, as distinct from recall.

Tables 7 and 8 indicate the spread of lesson elements across topics and the Year groups. (The data for the Year 5 lessons in these two tables include some double counting: two mental and oral starters and one plenary were coded as both recall and strategic.)

Tables 5 to 8 indicate a shift away from recall and towards procedural and strategic work as children progress through the primary school.

Work on recall in mental and oral starters: addition and subtraction

	Recall	Recall and procedural	Recall and strategic	**Total**
Year 1	9	3	0	**12**
Year 3	3	2	0	**5**
Year 5	2	1	2	**5**

Table 3

Work on recall in mental and oral starters: multiplication and division

	Recall	Recall and procedural	Recall and strategic	**Total**
Year 1	1	0	0	**1**
Year 3	5	2	0	**7**
Year 5	5	3	1	**9**

Table 4

Balance of procedural and strategic lesson elements: addition and subtraction

	Procedural			Strategic		
	Mental/oral	Main	Plenary	Mental/oral	Main	Plenary
Year 1	6	8	5	0	0	0
Year 3	4	9	7	2	0	0
Year 5	3	11	9	1	1	3

Table 5

Balance of procedural and strategic lesson elements: multiplication and division

	Procedural			Strategic		
	Mental/oral	Main	Plenary	Mental/oral	Main	Plenary
Year 1	–	–	–	–	–	–
Year 3	4	2	2	0	1	2
Year 5	4	6	5	3	0	0

Table 6

Balance of recall, procedural and strategic lesson elements: addition and subtraction*

	Recall	Procedural	Strategic
Year 1	14	19 (61%)	0
Year 3	6	20 (71%)	2 (7%)
Year 5	6	23 (74%)	5 (16%)

Table 7

Balance of recall, procedural and strategic lesson elements: multiplication and division*

	Recall	Procedural	Strategic
Year 1	n/a	n/a	n/a
Year 3	8	8 (47%)	3 (18%)
Year 5	9	15 (60%)	3 (12%)

Table 8

* Numbers in brackets are lesson elements expressed as a percentage of lessons with this subject focus observed.

This shift away from recall and towards procedural and strategic work appears more noticeable for lessons on addition and subtraction. It is also notable, within addition and subtraction lessons, that work that might be considered strategic increases in frequency as children get older; whether this effect is an indication of the kind of teacher that gets placed with different age groups or an effect of the Framework is not clear (generally more experienced teachers are placed with older children and in Year 2 where National Tests are taken). The picture is less clear for multiplication and division.

These figures raise a question about the extent to which learning procedures might be (or might be perceived to be) a prerequisite for working strategically.

Is it possible to ask Year 1 children to think about why they might want to count on, or why they might choose to count on from the largest number, before they have mastered counting on itself? Is it possible to ask Year 3 children to think about the size of jumps they might want to take on a number line, before they have mastered drawing jumps? It is our opinion that it is possible; the Framework, and the research it claims to draw on, would suggest that the two things (the skill and the thinking) should be developed in tandem.

Qualitative findings

Limitations of space prevent us from providing here much of the wealth of qualitative data we collected. We provide an example from a lesson seen in a Year 5 class. We have chosen this example as it demonstrates what we believe to be aspects of good practice and it illustrates the possibility of working on strategic approaches within the structure of the daily maths lesson. However, the lesson is not without its difficulties and, as such, also raises several issues.

Embedded (in italics) are our comments on the issues arising from the lesson together with an indication of themes (bold type) that are emerging from the analysis and which we intend to develop and report on in detail elsewhere.

The teacher, Elaine, was in her first year of teaching at the time of observation. She and the teacher of the parallel Year 5 class 'split' the group of children for mathematics lessons and Elaine took the 'less able' children from both classes. On the day that this observation took place there were 27 children and two learning support assistants (LSAs) in the room. The lesson occurred on a Wednesday; the lessons on the previous two days had been on mental strategies for addition and subtraction, and this lesson marked a move towards consideration of written methods; the final two lessons of the week continued the theme of written methods supported by mental calculation.

The lesson started with revision of place value, then considered some informal written methods for subtraction, after which point the children tried solving some word problems that involved the use of written addition and subtraction procedures. The incident that we describe in more detail below revolved around consideration of the usefulness of an informal written method; it took place during the mental and oral starter and was followed up indirectly in the plenary.

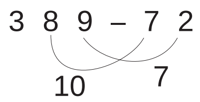
Critical incident

The teacher asked the children to consider the following calculation (written as below on a large whiteboard):

389 – 72

Before asking a child to solve the calculation, the teacher asked about the expected size of the answer – would it be bigger or smaller than 389? A child suggested ‘smaller’ and the teacher reinforced that the answer would be smaller as they were going to ‘take something away’. She further suggested that this was a way of knowing whether an answer is reasonable or not.

The teacher asked a child to work out the answer describing her thinking. She described using a partitioning strategy and the teacher, cueing her responses, drew up an image (given below) to accompany the description.



300 + 10 + 7 = 317

This brief description cannot fully convey the way in which the teacher elicited the explanation: she frequently interrupted the child with questions intended to make their explanation clearer, and the child’s response was, therefore, broken and staccato. This behaviour was typical, not only of this particular teacher’s interactions with pupils, but of all such interactions that we observed between the teachers and pupils. This lack of flow and high level of teacher control of the talk influenced the recording on the board, as the teachers constructed diagrams or pictures that accompanied the child’s description. What is noteworthy about this example is that the teacher’s construction of the diagram accurately followed and reflected what the child was telling her. In the majority of other such instances observed, the explanations and diagrams more closely accorded to what the teacher thought was important to model, rather than what the children actually described. **The issues of discussion and representations emerge as key themes.**

Having found a method and an answer for this calculation, the teacher shifted the children’s attention to the strategy itself and asked, ‘Do you remember what I said yesterday about using this method? When it’s the best time to use that one. Because sometimes it doesn’t quite work does it? Why doesn’t it sometimes work?’

There was no response to this, so she offered the children another calculation for them to consider:

321 – 97

The teacher asked the children whether this was easier or harder to work out than the previous example; once they had suggested that it was harder, she highlighted the 20 take away 90 by connecting them with a loop (as before), commenting, ‘Yeah – that’s the problem with that one. We’ve got to remember that we can take the big numbers away from the little numbers, but it’s more tricky to do 20 take away 90.’

Note that she did not tell the children that ‘you can’t take 90 from 20’; rather, ‘it is more tricky’. This was the only example we saw of a teacher accepting the possibility of subtracting a large number from a smaller one, although we saw many opportunities to do this. **There is an issue here (and elsewhere) of authority.**

As a response to his teacher’s comment that calculating 20 – 90 is more tricky than 90 – 20, a boy suggested that they swap the numbers around. ‘Change them round. Why don’t you do 397 take away 21?’ The teacher went with this suggestion, asking the class to indicate whether they thought that 397 – 21 would be the same as 321 – 97. While most knew the answers would be different, a few children believed they would be the same. To explore this, the teacher wrote up both calculations:

321 – 97 = 397 – 21 =

She asked a child to work out the answer to 397 – 21; this was successfully done, using the partitioning strategy applied above. The answer was recorded:

321 – 97 = 397 – 21 = 376

This was one of the rare examples that we observed, where the teacher took the time to explore, to good teaching effect, a common **misconception** *(in this case, that subtraction, like addition, is commutative (that a – b is the same as b – a). In other lessons with other teachers where such opportunities arose, at best the teacher tried to explain to the child why their suggestion was inappropriate, but often teachers simply told the child that something would not work or would not be correct, without any explanation.*

At this point, rather than complete the other calculation, the teacher drew the children’s attention to the conversation they had had at the beginning of this set of events, and asked them whether the answer to 321 – 97 would be bigger or smaller than 321. They said it would be smaller, and a child pointed out that 376 was greater than 321 (thus implying that this contradicted the conjecture that the answers to the two calculations would be the same). The teacher agreed and told them that, therefore, this could not be the answer. She concluded by saying, ‘Even without working this answer out, we know 376 can’t be right, because it’s bigger than 321, okay? So you can’t just swap all the numbers around because for subtraction it doesn’t work, okay?’

It is interesting that she did not finish all the calculations and that the choice she made was to connect it to a slightly separate issue – the idea of making sense. So she encouraged the children to reject the idea not because the answers did not agree but because this did not make sense in terms of what they knew about subtraction. **The main issue here is one of finality. Other issues raised include the nature of teacher resourcefulness, and establishing connections** *between aspects of the lesson and different mathematical concepts.*

The teacher returned to the issue of taking a larger number from a smaller one in the plenary, although she did not draw attention to the earlier part of the lesson. In the plenary, dice were used to generate numbers to be subtracted.

One example, 881 – 26, was calculated at a child's suggestion like this:

880 – 20 = 860

1 – 6 = – 5

860 – 5 = 855

Here a mixture of partitioning and knowledge about negative numbers are combined. The method followed a child claiming that 'you can't take 6 from 1, so you have to borrow'. Another child refuted this claim and declared that '1 take away 6 is negative 5'.

Again, we note the teacher's willingness to pursue this idea and the rarity of the use of such opportunities in the lessons that we observed. Given the rich potential of the strategic thinking involved in this method, the teacher's decision to use dice to generate numbers might have been unfortunate. It meant there was a lack of control over the numbers that needed to be subtracted and this was the only calculation where this issue of the need to take a larger number from a smaller number arose. **The issues of plenaries and planning questions are worth further exploration.**

Having been told that you can't swap the numbers around for subtraction, a child pointed out that you can for addition. The cases of multiplication and division were considered briefly, and the teacher explained that they would be returning to these ideas, but that today they were going to be looking at something else. The teacher used the word 'commutative' in her comments.

Here, again, the teacher demonstrated a willingness and ability to step aside from the main point of the lesson to help children to generalise and develop connections. It is also interesting, with regard to the term 'commutative', that she had interpreted the Strategy's explanation that 'children do not need to know this word' as 'they don't need to know it, but I can use it with them' – **this too is an issue of authority.**

Conclusions

This paper has focused largely on opportunity to learn mental calculation within the daily mathematics lesson prescribed by the Numeracy Strategy. We found that, while the three-part lesson was being used in all the classes that we observed, the balance of the lessons tended towards rather long mental/oral starters with brief (or omitted) plenaries. Within this structure, there was a predominance of recall and procedural work with limited attention to strategic thinking. From this, it seems to us that children were provided with the 'opportunity to learn' what might be termed 'mental arithmetic' (in the traditional sense of an emphasis on recall of answers and methods) rather than 'mental calculation' (in the spirit of the Numeracy Strategy).

Meeting our objectives

1. to review the understandings and interpretations of mental calculation that teachers are developing and that underpin their practices
2. to look at the range of practices in teaching mental calculation that teachers are developing in the light of the National Numeracy Strategy (NNS)
3. to examine the balance teachers attempt to achieve between children recalling number facts and developing strategies for effective mental calculation
4. to see how effective teachers are in going beyond the use of routines and artefacts (for example, number lines) demonstrated in NNS policy documents and training videos to appreciate the principles underpinning such techniques and so bring about shifts in their understanding and practice more generally

In line with other findings[7], the project has demonstrated that, while the amount of time and emphasis spent on mental work in mathematics has increased, the quality of such work remains a difficulty, with the emphasis largely placed on recall of answers or procedural methods. As with earlier findings[8], it would seem that teachers may have taken on board the surface features of the expected changes but have interpreted the expectations in the light of their existing beliefs and practices, rather then using them to effect 'deep' change.

5. to define what 'best practice' in mental calculation might look like
6. to review insights into pupils' responses to the range of practices

We did expect to see more 'good practice' in action and are disappointed at the limited evidence of this. While we can make suggestions for what good practice might look like in the light of what we actually saw, work still needs to be done to examine in greater detail pupils' responses to teaching that does focus on strategic approaches.

7. to examine the policy implications from all the above for developing training packages.

There are clear policy implications from our findings. Teachers need much more support in being helped to come to understand the principles behind calls for mental calculation and strategic approaches.

Further outcomes

Our current research findings have been made available through various channels: BEAM Education; conferences held by Psychology of Mathematics Education, the British Educational Research Association, and the British Society

7. *Teaching of calculation in primary schools* HMI 2002

8. *The National Numeracy Project* DfEE/HMI 1999

for Research into the Learning of Mathematics; research journals in mathematics education; and the professional journals read by teachers.

A report detailing our findings has been sent to all schools in the project, with a more detailed verbal report offered to the Phase 2 schools. Three of the project team (Askew, Bibby & Hodgen) are involved in writing material to be published by BEAM Education which will draw extensively on the findings of this project.

The project has raised further issues (flagged in the preceding pages) that we intend to follow up with more research. In particular, we hope to develop an intervention project to encourage good practice in this area and to examine children's responses to such practices.

References

Department for Education and Skills *Framework for teaching mathematics from Reception to Year 6,* DfES 1999

Department for Education and Skills *The National Numeracy Strategy*, DfES 1999

DfES/HMI *The National Numeracy Project: an HMI evaluation*, OFSTED/ HMSO 1999

HMI *Teaching of calculation in primary schools*, HMI 461 2002

Appendix 1

Data collection and analysis: Phase 1

Schools, teachers and observations
Phase 1

Name	Year	Years of experience	Phase 1 visits	Discussions /interviews
Southside LEA				
Chandler School				
Simon	Y1	NQT	1	1
Amelia	Y3	15+	1	1
Paula	Y3	6	1	1
Leyah	Y5	10+	1	1
Christine	Y5	10+	1	1
Southside LEA				
Millhone School				
Nina	Y1	NQT	1	1
Hannah	Y1	8	1	1
Sally	Y3	15+	1	1
Kim	Y3	6	1	1
Hazel	Y5	14	1	1
Steph	Y5	8	1	1
MarketTown LEA				
Spencer School				
Sarah	Y1	10+	1	1
Mandy	Y1	NQT	1	1
Chloe	Y3	10+	1	1
Matthew	Y5	NQT	1	1

Appendix 1

Appendix 2

Data collection and analysis: Phases 1 and 2

Schools, teachers and observations
Phases 1 and 2

Name	Year	Years of experience	Phase 1 visits	Phase 2 visits	Discussions /interviews
Southside LEA					
Marlow School					
Ella	Y1	NQT	1	3	4 / 2
Trupti	Y1	7	1		1 / 0
Lesley	Y3	7	1		1 / 0
Debbie	Y3	10+	1	3	4 / 2
Cath	Y5	3+	1		1 / 0
Lorna	Y5	10+	1	3	4 / 2
Northside LEA					
Grafton School					
Fatema	Y1	6	1		1 / 0
Diane	Y1	2	1	3	4 / 2
Lucy	Y3	20+	1	3	4 / 2
Estelle	Y3	10+, MaCo	1		1 / 0
Selina	Y5	10+	1	3	4 / 2
Susan	Y5	NQT	1		1 / 0
MarketTown LEA					
Parker School					
Natasha	Y1	NQT	1		1 / 0
Claire	Y1	NQT	1	2	2 / 2
Janice	Y3	20+, MaCo	1	3	4 / 2
Lisa	Y3	20+	1		1 / 0
Elaine	Y5	NQT*	1	3	4 / 2
Norma	Y5	6	1		1 / 0

Appendix 2

* maths BA Ed.

Other BEAM Education research papers

Raising attainment in primary number sense: from counting to strategy
by Mike Askew, Tamara Bibby and Margaret Brown

Making connections: effective use of numeracy
by Mike Askew

Teaching and learning primary mathematics: the impact of interactive whiteboards
by Penny Latham

Learning about numbers with patterns: using structured visual imagery (Numicon) to teach arithmetic
by Romey Tacon and Ruth Atkinson with Dr Tony Wing