research papers

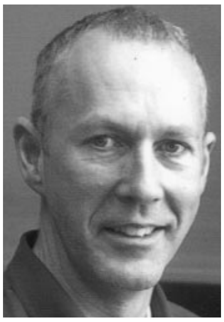
RES02

Making connections:

effective teaching of numeracy

Mike Askew
King's College London

Contents



Mike Askew

Mike Askew has taught in London primary schools and has written mathematics education materials for BEAM, Rigby, the Open University and the QCA. He is now a Reader at King's College London, where he directs the Doctor in Education (Ed.D), researches and writes. His research work has included prestigious commissions from the Teacher Training Agency (TTA), Nuffield and the Leverhulme Trust. Mike is a director of BEAM Education, and is closely involved in our product development and research initiatives. The research discussed here arose from the TTA-funded 'Effective Teachers of Numeracy' project.

BEAM Education

BEAM Education is a specialist mathematics education publisher, dedicated to promoting the teaching and learning of mathematics as interesting, challenging and enjoyable.

BEAM materials cater for Nursery and Reception through to Key Stage 3, and offer practical support and help with many of the current classroom concerns voiced by mathematics teachers.

BEAM services and materials include courses and in-service training for teachers; consultancy in mathematics education for government and other agencies; a comprehensive range of teaching accessories; and more than 80 publications for teachers of mathematics.

Published by BEAM Education

BEAM Education
Maze Workshops
72a Southgate Road
London N1 3JT

tel: 020 7684 3323
fax: 020 7684 3334
email: info@beam.co.uk
www.beam.co.uk

Three lessons

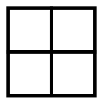
Lesson one

A class of seven-year-olds were organised in ability groups. Their teacher was working with a low-attaining group on doubling. The children spent a long time counting out individual cubes, fitting them together and re-counting them. The teacher set them a series of numbers to double, reiterating how to find the answer using the cubes, and went to join another group. The children were able to talk about what double 400 might be and quickly moved onto discussing doubling 3,000, 6 million and so on. After the lesson the teacher said that she was concerned that the children were not ready to be working with large numbers, particularly as no base-ten blocks had been available to help the children see that double 3,000 was 6,000.

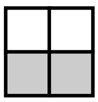
Lesson two

A class of nine-year-old children were working on equivalent fractions. The teacher drew a diagram on the board to demonstrate a way of converting a half into quarters. She explained that as quarters were the fraction they were converting to, the children would need to draw a rectangle divided into four equal parts.

$$\frac{1}{2} = \frac{\square}{4}$$



Since a half was required, two of these parts needed to be shaded.



‘So, a half is equivalent to two quarters’, explained the teacher. ‘On the other hand’, she continued, ‘we could just look at the numbers on the bottom of the fraction.’

$$\frac{1}{2} = \frac{\square}{4}$$

‘I have to multiply 2 (pointing to the 2 on the bottom of the half) by 2 to make 4 (pointing to the 4 on the board of the as yet denominator-free quarter fraction). So I multiply the 1 (pointing to the 1 on the top of the half) by 2 as well. We get two quarters, which is the same answer we got when we drew the diagram.’

$$\overset{\times 2}{\frac{1}{2} = \frac{\boxed{2}}{4}}_{\times 2}$$

The children were given a number of fractions to convert into equivalents and told that they could either use the diagram method or the multiplication method. They set to work on their own. Once individuals had completed a few examples using the diagram, the teacher moved around the class suggesting that it would be quicker to use the other method. The teacher helped children who were making errors by explaining both methods again. At the end of the lesson, the teacher went over the answers on the board, reminding the children of the rule: you multiply the top and the bottom by the same number.

Lesson three

The teacher of a class of ten-year-olds put a chart on the whiteboard that had columns for fractions, decimal fractions, percentages and ratios. One value had been entered in each row and the children were working in pairs to figure out how to convert values from one form of representation to another. Some of the values used equivalences that they were already familiar with, for example 0·5 or 25%. There were others that they did not know, for example $\frac{3}{8}$ or 0·65, and the children were having to work out their own methods of conversion. The pairs used a variety of methods, which they discussed with each other; they knew that they had to check their answers using a different method to the one that they had originally used. While they were working on the task, the teacher moved around listening to their explanations, taking notes on the different methods pairs were using and occasionally joining in the discussion. As they began to complete the task, the teacher brought the class together. Pairs were invited to come to the board to provide the answers missing from the chart and explain which method of calculation they had used. The teacher selected the pairs on the basis of the notes she had taken and included some children whose reasoning was inaccurate. The other children listened carefully to all the explanations offered. More efficient methods were suggested and errors were dealt with in a supportive manner, either by the teacher, or by other children. Finally, the class discussed the types of context in which the different representations would be used.

Introduction

Three very different lessons. Different teachers, different children, different mathematics. These examples illustrate ways in which teachers might set up lessons and some of the range of experiences from which children might learn. In this paper, I argue that differences like these may be based on the different beliefs that teachers have regarding the relationship between teaching and learning. I also examine research evidence that suggests that different beliefs may not only affect the style of teaching, but also pupils' learning.

Effective teachers of numeracy

Exploring teachers' beliefs, however informal or unarticulated, about the relationship between teaching and learning was one aspect of the 'Effective Teachers of Numeracy' project carried out at King's College in the mid-1990s by myself and colleagues Margaret Brown, Valerie Rhodes, Dylan Wiliam and David Johnson, and funded by the Teacher Training Agency[1].

1 The views expressed here are those of the author and should not be interpreted as representing the views of the Teacher Training Agency. Anyone wishing to read more about the project should refer to *Effective Teachers of Numeracy: Report of a study carried out for the Teacher Training Agency* King's college London (Askew, Brown, Rhodes, Wiliam & Johnson, 1997)

The principal aim of the 'Effective Teachers of Numeracy' project was 'to identify key factors which enable teachers to put effective teaching of numeracy into practice in the primary phase'. In developing this aim, we had first to define our terms. What exactly was meant by 'numeracy', and how were we to identify 'effective teaching'?

At the time of the project, the term numeracy was rarely used in education in the UK, and we could find no agreed definition. We therefore decided to adopt one that was broad enough to encompass the ability to calculate accurately but also went beyond that, to include a feel for number, and the ability to apply arithmetic. We came up with this:

> Numeracy is the ability to process, communicate and interpret numerical information in a variety of contexts.

With regard to effective teaching, many people in mathematics education – researchers, inspectors, teachers – would claim to know what 'good practice' in primary mathematics should look like. However, evidence linking teaching practices with learning outcomes is relatively limited. On the whole research into mathematics education in the UK separates findings on pupils' learning from findings about teaching. We decided, therefore, to base our definition of effective teaching on some measure of children's learning gains, rather than presumptions of 'good practice'. Once we had identified classes where children appeared to be learning more mathematics than in other, comparable, classes, we could go about exploring which practices appeared to be most effective in promoting this learning.

We chose to measure children's learning by looking at the gains for individual classes over part of a school year. Specially designed tests of numeracy were given to 90 classes, spanning ages five to eleven. The children were assessed near the beginning of the autumn term, and then again at the end of the spring term (the five-year-olds were only assessed on this second occasion).

On the basis of the children's test results, average gains were calculated for each class, providing an indicator of 'teacher effectiveness' for the 90 teachers in our project. We then set about looking for factors associated with these class gains.

We analysed questionnaires completed by each teacher; these included details about their qualifications, experience and teaching styles. To examine what might have made some teachers more effective than others, we looked at the relationship between the average gain in test results for each class and:

lesson organisation: whether the teachers taught the class as a whole, in groups, or set individual work

initial teaching qualifications: how the teachers had trained and the subjects in which they had specialised

mathematics qualifications: the level to which the teachers had studied mathematics as a subject in its own right

experience: how long the teachers had been teaching and the range of classes and ages taught

professional development: how much training the teachers had undergone since they qualified.

Some of our findings were surprising, as they challenge popularly held beliefs about what makes a teacher effective. For example, we found no association between average class gains and initial teaching qualifications, experience or whether the class was taught as a whole, in groups or individually.

Some of the teachers whose classes made the most gains did a lot of whole class teaching but so did some of the teachers with low average class gains. Similarly, group or individual work was used by teachers across the spread of results. The same published mathematics schemes were used by highly effective and comparatively much less effective teachers. The sort of qualifications that teachers had were not a good predictor of the average gains that their classes made, nor was their length of experience as a teacher.

Perhaps our most surprising finding was that there was a slight negative association between average class gains and the teachers' mathematics qualifications: the better qualified in mathematics the teachers were, the lower the average class gains. However, this finding should not be interpreted as indicating that one does not need to know much mathematics in order to teach it. A subset of eighteen of the 90 teachers was studied in greater depth, and this included interviewing them in detail about mathematics and their knowledge of it. Analysis of the responses to these interviews indicated that the teachers whose classes made great gains did in fact have a rich understanding of the mathematics that they taught. Taking the questionnaire and interview findings together suggests that simply being well qualified in mathematics may not be automatic proof that a teacher has the sort of mathematical knowledge required to teach the subject effectively.

In contrast, the amount of continuing professional development in mathematics education that teachers had undertaken was a better predictor of their effectiveness: there was a positive association between average class gains and the amount of in-service training in which the teachers had engaged.

If styles of classroom organisation and levels of mathematical qualification did not determine effectiveness, then what did? To better understand the factors that affect learning, we turned to the more detailed case study data that we had collected on eighteen teachers in the project. This data included notes taken from at least three observations of these eighteen teachers, followed by three extended interviews with each of them. The interviews covered several aspects of teaching; they looked at why the teachers had set up the lessons in the way they had, at their understanding of mathematics, at their views on why children were more, or less, successful in mathematics, and at what actions the teachers took as a result of their views. Through analysis of this interview and observation data, it became apparent that the main factor that seemed to make a difference was the teacher's belief about the nature of the relationship between teaching and learning.

Range of pupil attainment

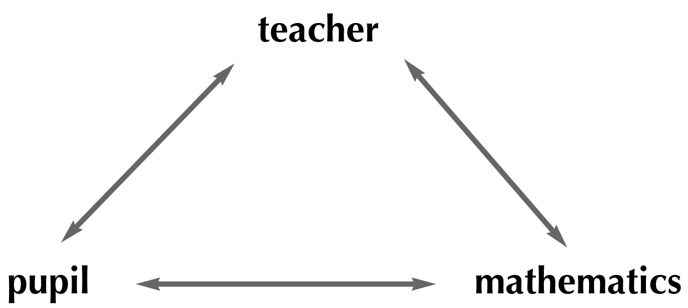
On the basis of the average gains made by each class, we put the teachers into three groups: 'highly effective', 'effective' and 'moderately effective'. In order to find out what contributed to the different gains made by different classes, we examined the information that we had about the case-study teachers in each category[2].

Highly effective	Effective	Moderately effective
Anne	Danielle	Beth
Alan	Dorothy	Brian
Alice	Eva	Cath
Barbara	Fay	David
Carole		Elizabeth
Faith		Erica

Table 1. Case-study teachers organised into three groups based on average class gain.

The teachers have been given pseudonyms to indicate which teachers worked at the same school. So, for example, Anne, Alan and Alice all taught in School A .

In examining the beliefs and understandings of these teachers, we looked at how they acted in class and how they talked about the relationships between teachers, pupils and mathematics.



From this, three orientations – clusters of beliefs about teaching and learning – emerged: discovery, transmission and connectionist.

'Discovery' orientation

Teachers who displayed evidence of the 'discovery' orientation towards teaching and learning placed particular emphasis on the pupil–mathematics link.

Pupil–teacher: The discovery-orientated teacher believes that it is the responsibility of the pupil to learn about mathematics. This means that pupil independence is valued above direct teaching.

2 Two case study teachers of Reception (Claire and Frances) are not included in the table. This was because their classes were tested only once, so gains could not be calculated to measure their effectiveness.

Emphasis on the pupil as an independent learner presupposes that what is to be learned has not been taught. Having to explicitly teach something is seen as less successful than the pupils learning it for themselves. Alternatively, if a pupil fails to learn something, then this may be seen as the result of a lack of readiness rather than as the result of inappropriate teaching methods.

Pupil–mathematics: Within the discovery orientation, prior understandings were seen as important determinants of what pupils are 'ready' to learn. Teachers with this orientation spoke as if there were a natural order in which pupils develop concepts, and so progress, and rates of learning were determined by this more than by teaching practices. Teachers displaying a discovery orientation focused on affective aspects of learning mathematics: to learn effectively pupils needed be motivated and able to work independently. Activities were therefore structured around these aims and justified on the basis of being enjoyable and set up to be 'fun'. A heavy emphasis was placed on practical work, with the conventions of the subject subordinated to understanding. Lesson one at the beginning of this paper illustrates this. The teacher expected the children to use cubes to do the doubling and find the answers, even though in conversation with the children it became clear that they could use their knowledge of simple doubles, for example, double 4, to double numbers like 400 or 4000.

Teacher–mathematics: For the discovery-oriented teacher, their role is primarily to set up activities and learning experiences that will facilitate pupils finding out about mathematics independently. So their main focus when thinking about teaching mathematics is to find ways to engage pupil interest, rather than to consider the nature of the mathematics to be learned.

This quote from one of the teachers in the study largely sums up this orientation:

> *Well, we try and make sure that any work we are giving them is appropriate to their ability, sort of get them to the concepts at the appropriate times. [We try] to encourage their independence in choosing apparatus they may use when they are doing different bits of maths work, [and] arrange the classrooms so the children can work without an adult. The child can go and take the tools necessary to do the work, and already make practical use of the classroom, so the children can develop in that way.*

'Transmission' orientation

Teachers who displayed evidence of the 'transmission' orientation towards teaching and learning placed particular emphasis on the teacher–mathematics link.

Pupil–teacher: The transmission orientation is marked by a clear separation of teaching and learning, with the emphasis on teaching. Teachers disposed towards this orientation see pupils as dependent upon the teacher for gaining access to mathematics. The transmission-oriented teacher regards themself as primarily responsible for the learning. The pupil's role in the class is subordinate to that of the teacher. Lesson two, at the beginning of this paper, is typical of the ‘transmission’ style. Here, the emphasis is on manipulating symbols and teaching the same techniques again if the children did not appear to understand the first time.

This quote from one of the teachers about what makes some children more, or less, successful, sums up this position:

> *A lot of them learn by rote ... (some)one who needs extra help, I will stand behind him when he is doing it and actually work with him for a long long time ... so they (the weakest) get a lot more of my help. Really I teach it by rote ... These two are very weak ... They have to learn it by rote...if the child gets a low mark it’s probably my fault not the child’s.*

Pupil–mathematics: Within a transmission orientation, mathematics is seen to be governed by rules and procedures. Mathematics is a metaphorical set of objects to be passed on, a body of knowledge to be committed to memory. The transmission orientation focuses on learning in terms of the pupils’ ability to retain mathematical ideas, the evidence for which comes from whether the examples worked through are correctly done. A heavy emphasis is placed on the symbolic and notational aspects of mathematics – pupils laying out work ‘correctly’, in other words in line with the accepted conventions, is a major part of the practice of doing mathematics and a main source of assessment. Pupils are also required to follow mathematical procedures: the answers must be correct, and so, equally, must the methods of solution.

Teacher–mathematics: Planning for teaching within a transmission orientation means attending in the main to what is to be taught, not what has been learned. This means that the teacher has to know how to break the curriculum down into step-by-step pieces that can be taught sequentially. The prior understanding that pupils have is seen to be of little interest. This was illustrated during Lesson two when one girl interpreted the diagram on the board differently from how the teacher intended. The interpretation that the girl made was that in order, say, to convert two thirds into sixths she had to draw a block of six squares and shade in three for a third and then the second three for the two thirds. When the teacher noticed this girl’s working, rather than asking the girl to explain her method, she said, ‘I think you’ve done enough shading in. Do the rest the other way.’

‘Connectionist’ orientation

The connectionist orientation encompasses a view of teaching and learning that attempts to reconcile ‘teach’ and ‘learn’, rather than treat them as opposites. The connectionist-oriented teacher focuses on all three of the bonds in the teaching triad: pupil–teacher, pupil–mathematics and teacher–mathematics.

Pupil–teacher: The connectionist-oriented teacher aims to develop pupils’ learning, while acknowledging the role that teaching plays in this. This does not simply mean teachers paying attention to learners. The connectionist-orientated teacher has a strong sense of themself being a learner too, constantly learning about their pupils. Interest in what pupils have previously learned, how to make sense of pupils' interpretations of the lessons and how this might be taken into account in planning and teaching, is typical within the connectionist orientation. One of the teachers summed up the importance of establishing good relationships with the children:

> *I can't work with the child unless I am able to have some toehold as to what the child's strengths and weaknesses are. I can test a child in a formal setting, but I find it so important to be able to communicate with the child on a one-to-one level and to have the child be open and honest with me.*

Pupil–mathematics: The connectionist orientation draws on mathematics as a network of connections. The connections between different topics are common features of lessons taught within this orientation. Lesson three is typical of this, in the way that fractions, decimals, ratios and proportions were not treated as separate topics but related to each other. Other examples would include lessons where addition and subtraction or multiplication and division were taught together. Activities represent the complexity of mathematics, rather than fragmenting the curriculum into discrete topics. Multiple representations of mathematical ideas are used. For example, in a lesson on place value, we observed that the children had to move between expressing numbers in symbols, with base-ten blocks, and placing counters in two hoops designated as 'tens' and 'ones'.

The importance of discussion to establish shared meanings is implicit in the connectionist orientation. This is shown through interactions with pupils where understandings are shared, and pupils are given time to explain their understandings, while the teacher still provides alternative methods and explanations. As one of the teachers put it:

> *Children are so mysterious .... You just need to talk with them and for them to explain the mechanics, the thinking of what they are doing and don't make any assumptions ... I just say, don't worry I'll show you a different way tomorrow. I will go out and rack my brains ... or I might ask a child. I ask the children to explain to each other as well.*

Teacher–mathematics: Planning for teaching within a connectionist orientation means attending to both what has been learned as well as to what is to be taught. This means the teacher has to know not only how pupils learn mathematics in general, and the understandings of the particular pupils they are teaching, but also effective activities and ways to explain.

Orientations and pupil learning

The connectionist, transmission and discovery orientations are ideal types: no single teacher is likely to hold a set of beliefs or practices that precisely matches those set out within each orientation. Teachers, like anyone, will develop personal preferences that draw on a variety of beliefs or practices.

However, analysis of our data revealed that some teachers were more predisposed to talk and behave in ways that fitted with one orientation over the others. In particular, Anne, Alan, Barbara, Carole, Claire, and Faith, all displayed characteristics indicating a high level of orientation towards the connectionist view. On the other hand, Brian and David both displayed strong discovery orientations, while Beth, Cath and Elizabeth were characterised as transmission-orientated teachers.

Other case-study teachers displayed less distinct allegiance to one of the three orientations. They held sets of beliefs that drew from one or more of the orientations. For example, one teacher had strong connectionist beliefs about the nature of being a numerate child, but in practice displayed a transmission orientation towards how best to teach children to become numerate.

Comparing the grouping of the teachers into the three orientations alongside the previous classification of the teacher's relatively high, medium or low average class gain scores suggests that there may be a relationship between pupil learning outcomes and teacher orientations.

	Highly effective	**Effective**	**Moderately effective**
Strongly Connectionist	Anne Alan Barbara Carole Faith		
Strongly transmission			Beth Cath Elizabeth
Strongly discovery			Brian David
No strong orientation	Alice	Danielle Dorothy Eva Fay	Erica

Table 2. Comparison of teacher orientation with average class gain

Discussion

At the time of writing, teachers in England are being encouraged to adopt a particular style of lesson, one that has three distinct parts: an oral and mental starter, a main teaching section and a plenary. Throughout the lesson there is meant to be plenty of interactive whole class teaching. While there is no doubt that this has increased access to mathematics for many primary school children, it is also clear that there are still wide variations in their learning. I suggest that we need to look beyond surface features of lessons to understand why different teachers have different impacts. Examining orientations towards teaching mathematics can help us understand why practices that have surface similarities may result in different learner outcomes.

For example, while all the teachers in our study employed some whole class question and answer sessions, the nature of the interactions with children within such sessions varied according to the teacher's orientation. The transmission-oriented teachers tended only to focus on correct answers, based on methods that they had previously taught. Any incorrect answers were not explored. The discovery-oriented teachers were interested in methods as well as answers, and valued the fact that the children produced a range of methods, but their discussion did not extend to considering whether or not some methods were more, or less, efficient than others. The connectionist-oriented teachers also valued the range of methods that the children came up with, but they also discussed the relative merits of different methods, and, when appropriate, suggest further methods that the children had not thought of.

Expecting teachers to adopt new practices, like 'the three part lesson' may result either in the practices being adapted to fit with existing beliefs, or in limited uptake of the practices themselves. As other research on developing teaching has demonstrated, exhorting teachers to adopt particular practices, without helping them develop a deep understanding of the principles behind these practices, does not in itself lead to raised standards (Alexander, 1992). In a current project at King's we are exploring this issue in more detail[3]. An example taken from a observation of a lesson from this programme illustrates the difficulty of changing beliefs and practices.

In this lesson, the children were working on simple shopping problems, presented orally by the teacher, and with each child writing their solution on an individual whiteboard to hold up and show the answer. The children were getting the majority of the answers correct and the teacher invited two or three to explain how they arrived at their answer. However, the discussion did not include exploring whether any particular methods were more efficient than others. In fact, the children seemed to be treating the discussion as a challenge to see who could come up with the most unusual method. As the questions got more difficult, and several of the children began to show incorrect answers, the teacher ceased to ask how they had worked them out and instead went on to show how to use paper-and-pencil methods to work out the answers. This resulted in more of the children making mistakes, as they tried to use the teacher's method rather than one that made sense to them personally.

Teachers may find it helpful to examine their belief systems and think about where they stand in relation to these three orientations. In a sense, the connectionist approach is not a complete contrast to the other two but embodies the best of both of them in its acknowledgement of the role of both teacher and pupil in lessons. Teachers may, therefore, need to address different issues according to their beliefs: the transmission-oriented teacher may want to consider the attention given to pupil understandings, while the discovery-oriented teacher may need to examine beliefs about the role of the teacher.

Anna Sfard (1998) suggests that there are two main ways in which we talk about, and consequently think about, learning: learning as a process of acquisition, and learning as a process of participation. There is much talk about teaching that assumes an acquisition model: delivering the curriculum, raising standards, whether or not pupils have 'got it'.

3 The Leverhulme Numeracy Research Programme

While not wishing to dispute the fact that the overall aim of teaching should be that pupils have acquired some mathematical knowledge, the sort of lesson that they have participated in on the way to acquiring that knowledge will have a dramatic impact on the sort of knowledge they acquire. Perhaps the thing that most distinguished our connectionist-orientated teachers was the ways in which they tried to make their classrooms 'communities of learners', where everyone learned from everyone else (Rogoff, 1995). One of the connectionist-orientated teachers expressed it in this way:

> *I think that as each year goes on, you learn more, and you discover that you need to know more, as each year goes on, rather than when you start. So I think that your learning curve is disproportionate, in that you will think it would get easier, but in fact as each year goes on you see different ways of doing things that will make [your teaching] better, in different ways of extending things. I think that you are learning each year as you go on, and there is more and more to learn.*

References

Alexander, R (1992) *Policy and Practice in Primary Education* Routledge, London

Askew, M, Brown, M, Rhodes, V, Wiliam, D, & Johnson, D (1997) *Effective Teachers of Numeracy: Report of a study carried out for the Teacher Training Agency* King's College London

Leverhulme Numeracy Research Programme, the. For more details see www.leverhulme.org.uk.

Rogoff, B (1995) *Evaluating development in the process of participation: theory, methods, and practice building on each other* from E Amsel & A Renninger (Eds.) *Change and development: Issues of theory, application and method* Lawrence Erlbaum, Hillsdale, NJ, USA

Sfard, A (1998) On two metaphors for learning and the dangers of choosing just one *Educational Researcher*, 27(2), 4-13